

6. Estimates will vary; for example:

$$\begin{aligned} \text{a) } \sqrt{\frac{8}{10}} &= \sqrt{\frac{80}{100}} \text{ and } \\ \sqrt{\frac{80}{100}} &\doteq \sqrt{\frac{81}{100}} \\ &= \sqrt{\frac{9}{10} \times \frac{9}{10}} \\ &= \frac{9}{10} \\ \text{So, } \sqrt{\frac{8}{10}} &\doteq \frac{9}{10}. \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{\frac{17}{5}} &= \sqrt{\frac{85}{25}} \\ \text{In the fraction } \frac{85}{25}, & \text{ 85 is close to the perfect } \\ & \text{square 81.} \\ \text{So, } \sqrt{\frac{85}{25}} & \text{ is close to } \sqrt{\frac{81}{25}}. \\ \sqrt{\frac{85}{25}} &\doteq \sqrt{\frac{81}{25}} \\ &= \sqrt{\frac{9}{5} \times \frac{9}{5}} \\ &= \frac{9}{5} \\ \text{So, } \sqrt{\frac{17}{5}} &\doteq \frac{9}{5}. \end{aligned}$$

$$\begin{aligned} \text{c) In the fraction } \frac{7}{13}, & \text{ 7 is close to the perfect } \\ & \text{square 9, and 13 is close to the perfect square } \\ & \text{16.} \\ \text{So, } \sqrt{\frac{7}{13}} &\doteq \sqrt{\frac{9}{16}} \\ \sqrt{\frac{9}{16}} &= \frac{3}{4} \\ \text{So, } \sqrt{\frac{7}{13}} &\doteq \frac{3}{4}. \\ \text{Or, another way:} \\ \sqrt{\frac{7}{13}} &\doteq \sqrt{\frac{49}{100}} \text{ and } \sqrt{\frac{49}{100}} = \frac{7}{10}. \\ \text{So, } \sqrt{\frac{7}{13}} &\doteq \frac{7}{10}. \end{aligned}$$

d) In the fraction $\frac{29}{6}$, 29 is close to the perfect square 25, and 6 is close to the perfect square 4.

$$\text{So, } \sqrt{\frac{29}{6}} \doteq \sqrt{\frac{25}{4}}$$

$$\sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$\text{So, } \sqrt{\frac{29}{6}} \doteq \frac{5}{2}.$$

Or, another way:

$$\sqrt{\frac{29}{6}} \doteq \sqrt{\frac{169}{36}} \text{ and } \sqrt{\frac{169}{36}} = \frac{13}{6}.$$

$$\text{So, } \sqrt{\frac{29}{6}} \doteq \frac{13}{6}.$$

10. Answers may vary.

a) $3^2 = 9$ and $4^2 = 16$, so any number between 9 and 16 has a square root between 3 and 4. Two decimals between 9 and 16 are 10.24 and 12.25.

$$\sqrt{10.24} = 3.2 \text{ and } \sqrt{12.25} = 3.5$$

b) $7^2 = 49$ and $8^2 = 64$, so any number between 49 and 64 has a square root between 7 and 8. Two decimals between 49 and 64 are 50.41 and 59.29.

$$\sqrt{50.41} = 7.1 \text{ and } \sqrt{59.29} = 7.7$$

c) $12^2 = 144$ and $13^2 = 169$, so any number between 144 and 169 has a square root between 12 and 13. Two decimals between 144 and 169 are 158.36 and 166.41.

$$\sqrt{158.36} \doteq 12.6 \text{ and } \sqrt{166.41} = 12.9$$

d) $1.5^2 = 2.25$ and $2.5^2 = 6.25$, so any number between 2.25 and 6.25 has a square root between 1.5 and 2.5. Two decimals between 2.25 and 6.25 are 3.0 and 3.5.

$$\sqrt{3.0} \doteq 1.7 \text{ and } \sqrt{3.5} \doteq 1.9$$

e) $4.5^2 = 20.25$ and $5.5^2 = 30.25$, so any number between 20.25 and 30.25 has a square root between 4.5 and 5.5. Two decimals between 20.25 and 30.25 are 22.09 and 29.16.

$$\sqrt{22.09} = 4.7 \text{ and } \sqrt{29.16} = 5.4$$

11. I used benchmarks to estimate the value of each square root, because they provided a small range of numbers to work with.

a) 4.5 is between the perfect squares 4 and 9, and closer to 4.

So, $\sqrt{4.5}$ is between 2 and 3, and closer to 2.

Estimate $\sqrt{4.5}$ as 2.1.

To check, evaluate: $2.1^2 = 4.41$

This is very close to 4.5, so $\sqrt{4.5} \doteq 2.1$.

b) $\frac{17}{2} = 8.5$

8.5 is between the perfect squares 4 and 9, and closer to 9.

So, $\sqrt{\frac{17}{2}}$ is between 2 and 3, and closer to 3.

Estimate $\sqrt{\frac{17}{2}}$ as 2.9.

To check, evaluate: $2.9^2 = 8.41$, which is very close to 8.5.

So, $\sqrt{\frac{17}{2}} \doteq 2.9$

c) 0.15 is between the perfect squares 0.09 and 0.16, and closer to 0.16.

So, $\sqrt{0.15}$ is between 0.3 and 0.4, and closer to 0.4.

Estimate $\sqrt{0.15}$ as 0.39.

To check, evaluate: $0.39^2 = 0.1521$, which is very close to 0.15.

So, $\sqrt{0.15} \doteq 0.39$

d) In the fraction $\frac{10}{41}$, 10 is close to the perfect

square 9, and 41 is close to the perfect square 36.

So, $\sqrt{\frac{10}{41}} \doteq \sqrt{\frac{9}{36}}$

$\sqrt{\frac{9}{36}} = \frac{3}{6}$, or $\frac{1}{2}$

So, $\sqrt{\frac{10}{41}} \doteq \frac{1}{2}$, or 0.5

e) 0.7 is between the perfect squares 0.64 and 0.81, and closer to 0.64.

So, $\sqrt{0.7}$ is between 0.8 and 0.9, and closer to 0.8.

Estimate $\sqrt{0.7}$ as 0.84.

To check, evaluate: $0.84^2 = 0.7056$, which is very close to 0.7.

So, $\sqrt{0.7} \doteq 0.84$

f) In the fraction $\frac{8}{45}$, 8 is close to the perfect

square 9, and 45 is close to the perfect square 49.

So, $\sqrt{\frac{8}{45}} \doteq \sqrt{\frac{9}{49}}$

$$\sqrt{\frac{9}{49}} = \frac{3}{7}$$

So, $\sqrt{\frac{8}{45}} \doteq \frac{3}{7}$, or about 0.4

g) 0.05 is between the perfect squares 0.04 and 0.09, and closer to 0.04.

So, $\sqrt{0.05}$ is between 0.2 and 0.3, and closer to 0.2.

Estimate $\sqrt{0.05}$ as 0.22.

To check, evaluate: $0.22^2 = 0.0484$, which is very close to 0.05.

So, $\sqrt{0.05} \doteq 0.22$

h) In the fraction $\frac{90}{19}$, 90 is close to the perfect

square 81, and 19 is close to the perfect square 16.

So, $\sqrt{\frac{90}{19}} \doteq \sqrt{\frac{81}{16}}$

$\sqrt{\frac{81}{16}} = \frac{9}{4}$

So, $\sqrt{\frac{90}{19}} \doteq \frac{9}{4}$

12. Estimates may vary. I used benchmarks.

a) In the fraction $\frac{3}{8}$, 3 is close to the perfect square

4, and 8 is close to the perfect square 9.

So, $\sqrt{\frac{3}{8}} \doteq \sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{3}{8}} \doteq \frac{2}{3}$, or about 0.7 to the nearest tenth

b) In the fraction $\frac{5}{12}$, 5 is close to the perfect

square 4, and 12 is close to the perfect square 9.

So, $\sqrt{\frac{5}{12}} \doteq \sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{5}{12}} \doteq \frac{2}{3}$, or about 0.7 to the nearest tenth

c) In the fraction $\frac{13}{4}$, 13 is close to the perfect

square 16, and 4 is a perfect square.

$$\text{So, } \sqrt{\frac{13}{4}} \doteq \sqrt{\frac{16}{4}}$$

$$\sqrt{\frac{16}{4}} = 2$$

$$\text{So, } \sqrt{\frac{13}{4}} \doteq 2.0$$

- d) I wrote the fraction as a decimal, then used benchmarks.

$$\frac{25}{3} \doteq 8.3$$

The closest perfect squares on either side of 8.3 are 4 and 9.

$$\sqrt{4} = 2 \text{ and } \sqrt{9} = 3$$

8.3 is closer to 9, so choose 2.9 as a possible estimate for a square root.

To check, evaluate: $2.9^2 = 8.41$, which is close to 8.3.

$$\text{So, } \sqrt{\frac{25}{3}} \doteq 2.9$$

13. Write the Pythagorean Theorem in each right triangle to determine the unknown length.

$$\begin{aligned} \text{a) } h^2 &= 1.2^2 + 0.5^2 \\ &= 1.44 + 0.25 \\ &= 1.69 \\ h &= \sqrt{1.69} \\ &= 1.3 \end{aligned}$$

The unknown length h is 1.3 cm.

$$\begin{aligned} \text{b) } h^2 &= 1.5^2 + 2.2^2 \\ &= 2.25 + 4.84 \\ &= 7.09 \\ h &= \sqrt{7.09} \\ &\doteq 2.663 \end{aligned}$$

The unknown length h is about 2.7 cm.

$$\begin{aligned} \text{c) } s^2 + 2.8^2 &= 5.6^2 \\ s^2 &= 5.6^2 - 2.8^2 \\ &= 31.36 - 7.84 \\ &= 23.52 \\ s &= \sqrt{23.52} \\ &\doteq 4.85 \end{aligned}$$

The unknown length s is about 4.9 cm.

$$\begin{aligned} \text{d) } s^2 + 2.4^2 &= 2.5^2 \\ s^2 &= 2.5^2 - 2.4^2 \\ &= 6.25 - 5.76 \\ &= 0.49 \\ s &= \sqrt{0.49} \end{aligned}$$

$$= 0.7$$

The unknown length s is 0.7 cm.

18. a) The numbers greater than 1 have square roots that are less than the given number.

For example: $\sqrt{2} \doteq 1.4142$, $\sqrt{4} = 2$, $\sqrt{10} \doteq 3.1623$, $\sqrt{100} = 10$

- b) The number must be 0 or 1 for its square root to be equal to the number.

$$\sqrt{0} = 0, \sqrt{1} = 1$$

- c) The numbers less than 1 have square roots that are greater than the given number.

For example: $\sqrt{0.1} \doteq 0.3162$, $\sqrt{0.01} = 0.1$, $\sqrt{0.09} = 0.3$, $\sqrt{0.025} \doteq 0.1581$

19. Answers will vary. For example:

- a) The number with a square root of 1 is 1. So, any number between 0 and 1 has a square root between 0 and 1. For example: 0.64

$\sqrt{0.64}$ is between 0 and 1.

- b) The number with a square root of 1.5 is 2.25.

The number with a square root of 2 is 4.

So, any number between 2.25 and 4 has a square root between 1.5 and 2. For example: 3

$\sqrt{3} \doteq 1.7321$ is between 1.5 and 2.

- c) Write $\frac{1}{2}$ and $\frac{3}{4}$ as decimals:

$$\frac{1}{2} = 0.5 \text{ and } \frac{3}{4} = 0.75$$

The number with a square root of 0.5 is 0.25. The number with a square root of 0.75 is 0.5625. So, any number between 0.25 and 0.5625 has a square root between $\frac{1}{2}$ and $\frac{3}{4}$.

For example: 0.4, or $\frac{4}{10}$

$$\begin{aligned} \sqrt{0.4} &= \sqrt{\frac{4}{10}} \\ &\doteq 0.63 \end{aligned}$$

So, $\sqrt{0.4}$ is between $\frac{1}{2}$ and $\frac{3}{4}$

- d) Write $3\frac{3}{4}$ as a decimal:

$$3\frac{3}{4} = 3.75$$

The number with a square root of 3.75 is 14.0625.

The number with a square root of 4 is 16.

So, any number between 14.0625 and 16 has a

square root between $3\frac{3}{4}$ and 5.

For example: 15

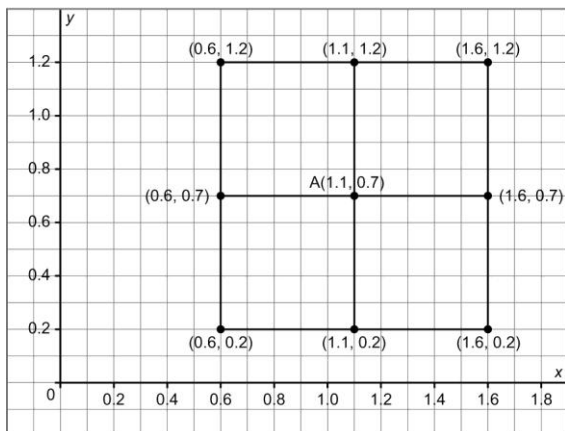
$\sqrt{15} \div 3.873$ is between $3\frac{3}{4}$ and 4.

$$\begin{aligned} 20. \text{ a) } AB^2 &= 0.5^2 + 1.75^2 \\ &= 0.25 + 3.0625 \\ &= 3.3125 \\ AB &= \sqrt{3.3125} \\ &\div 1.82 \\ AB &\text{ is about } 1.82 \text{ km.} \end{aligned}$$

$$\begin{aligned} \text{b) } AB^2 &= 1.25^2 + 2^2 \\ &= 1.5625 + 4 \\ &= 5.5625 \\ AB &= \sqrt{5.5625} \\ &\div 2.3585 \\ AB &\text{ is about } 2.36 \text{ km} \end{aligned}$$

$$\begin{aligned} 22. \quad \sqrt{0.6} &\div 0.775 \text{ and } \sqrt{0.61} \div 0.781 \\ \text{So, all numbers between } 0.775 \text{ and } 0.781 &\text{ have} \\ \text{squares between } 0.6 \text{ and } 0.61. \\ \text{For example: } 0.776^2 &\div 0.6022, 0.779^2 \div 0.6068, \\ \text{and } 0.78^2 &= 0.6084, \text{ which are all between } 0.6 \text{ and} \\ 0.61. \end{aligned}$$

23. The square has area 0.25 square units.
Take the square root of its area to determine the side length.
 $\sqrt{0.25} = 0.5$
So, the side length of the square is 0.5 units.
The scale is 1 grid square represents 0.1 units. So, 0.5 units represent 5 grid squares.
Since one vertex of the square is (1.1, 0.7), the other vertices of one possible square are:
(1.1, 0.2), (0.6, 0.2), and (0.6, 0.7).



It is possible to get other vertices by considering different positions of the square:

(1.1, 0.2), (1.6, 0.2), and (1.6, 0.7) or
(1.6, 0.7), (1.6, 1.2), and (1.1, 1.2) or
(1.1, 1.2), (0.6, 1.2), and (0.6, 0.7).

24. a) The area of the original photograph, in square centimetres, is $(5.5)^2$, or 30.25 cm^2 .
The area of the larger photograph is: $30.25 \text{ cm}^2 \times 2 = 60.5 \text{ cm}^2$
To determine the side length of the square, take the square root of the area of the square:
 $\sqrt{60.5} \div 7.8$
So, the side length of the larger photograph is approximately 7.8 cm.

- b) For example, a square with side length 5.5 cm has an area of $(5.5)^2$, or 30.25 cm^2 .
Double the side length of the square: $5.5 \text{ cm} \times 2 = 11 \text{ cm}$
A square with side length 11 cm has an area of $(11)^2$, or 121 cm^2 , which is $30.25 \text{ cm}^2 \times 4$.
So, doubling the side length of a square does not double the area; it increases the area by a factor of 4.